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A field theoretic view of atom-atom collision

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The process of atom-atom collision is treated in a field theoretic way. The non-relativistic interaction potential is shown to be consequence of the field interaction.

Introduction

Excitation or ionization of an atom by the impact of another atom is a phenomen on which has a basic importance in many physical processes. There are usually two kinds of such processes, (1) when the striking atom does not change the state, called a collision of the first kind, and (2) when the initial state of the striking atom also gets changed, called a collision of the second kind. Theoretical explanation of these processes are usually given by the help of usual Born approximation. Bates & Griffling (1953) has made an exhaustive treatment of these kind of events. The main defect of their treatment is that they cannot justify the use of the interaction potential which is written in a phenomenological way. Our field theoretic treatment of the bound state gives rise to these interactions, from the usual interaction between, electron-proton field and electromagnetic field in the form $J_\mu A_\mu$, J_μ being the current corresponding to the nucleon.

Process of the First Kind

We consider the collision of two atoms consisting of two nucleui of charges Z_1, Z_2 respectively (being considered as Fermions) each surrounded by a single electron. The interaction Hamiltonian for the whole system is written as.

$$H_{\text{int}} = J_\mu^1 A_\mu + J_\mu^2 A_\mu + J_\mu^e A_\mu \tag{1}$$

where $J_\mu^{1,2,e}$ are the currents due to the two nucleui and an electron. The initial and final state vectors are written (Roy 1960) as

$$| \psi_i > = \int g_1^1 (\mu,\mu') a^+ (\mu) A^+ (\mu') g_2^0 (\sigma,\sigma') a^+ (\sigma) B^+ (\sigma') | 0 > d_n d_p' d\sigma d\sigma' \tag{2}$$

$$| \psi_f > = \int g_1^2(\tau,\tau') a^+ (\tau) A^+ (\tau') g_2^e (\lambda,\lambda') a^+ (\lambda) B^+ (\lambda') | 0 > d\tau d\tau' d\lambda d\lambda'$$

where a^+, A^+, B^+ are creation operators for the electron and the two nucleui. The matrix element of the process is given by (Schweber 1964).

$$M_{fi} = \frac{1}{2!} \int < \psi_f \left| H_{\text{int}} H_{\text{int}} \right| \psi_i > \tag{3}$$

$H_{int}H_{int}$ contains different combinations of the 3 terms in (1), one of which is seen to be $e^2(\bar{\psi}\nu_\mu A_\mu \psi\, \bar{\psi}\nu_\nu A_\nu \psi)$ (4)

Here ψ'_s have the usual expansion, $\psi(x) = \int a(p)e^{ip\cdot x}\, d^3p$ (5)

Spin of the particle being neglected, substituting for $|\psi_i>$, $|\psi_f>$ we have the following form of M_{fi},

$$M_{fi} = \frac{e^2}{2!}\int <0 \mid B(\lambda')\, a(\lambda)\, g_2{}^{0*}(\lambda,\lambda')\, A(\tau')\, a(\tau) g_1{}^{i}(\tau,\tau')$$
$$a^+(p)\, a(p')\, a^+(q)\, a(q')\, g_1{}^{i}(\mu,\mu')\, a^+(\mu)\, A^+(\mu')\, g_2{}^{e}(\sigma,\sigma')$$
$$a^+(\sigma)\, B^+(\sigma') \mid 0> \frac{\delta(p' - p + q' - q)}{(q' - q)^2} \qquad (6)$$

Since we are interested only in the non-relativistic region, we have separated out the retarded interaction from the Coloumb by an extension of the technique of Feynmann (1949) which is due to Intemann & Pollock (1967) and the Coloumb part only remains in the NR limit.

The vacuum expectation in (6) gives different combinations of delta functions, one such term is

$$\delta(p' - \mu)\delta(p - \lambda)\delta(\tau - \sigma)\delta(q' - \sigma')\delta(q - \tau')\delta(\lambda' - \mu') \qquad (7)$$

This when combined with, the $g(k,k)$'s occuring in (6) and integrated over the arguments of delta functions gives one term of M_{fi}, which is

$$M_{fi}(ee) = \frac{1}{(Q - P)^2}\int d\lambda \phi_1{}^{i}(\lambda, P - \lambda)\, \phi_1{}^{i}(\lambda - P + \theta, P - \lambda)$$
$$\int \phi_2{}^{e}(\sigma, P' - \sigma) \times \phi_2{}^{0}(\sigma, Q' - \sigma)\, d\sigma \qquad (8)$$

where we have used the following expression for (gk,k) expressing the breaking up of the centre of mass and relative motion,

$$g(k,k') = \phi(k,k')\, \delta(k + k' - P) \qquad (9)$$

If we now use the inverse Fourier transforms of the ϕ's (the bound state wave functions) it can be easily seen that (8) boils down to the following form

$$M_{fi}(e,e) = \int \phi_2{}^{*0}(r_b)\, V \phi_2{}^{e}(r_b) \qquad (10)$$

where, $V = \int \phi_1{}^{i}(r_a) \frac{e^2}{|R + \bar{r}_b - \bar{r}_a|} \phi_1{}^{*s}(r_a)\, dr_a.$ (11)

When, all the terms of H_{int} H_{int} are treated in the same manner, then total matrix element of the process is seen to be,

$$M_{fi} = M_{fi}(ee) + M_{fi}(P_1,P_2)\, M_{fi}(eP_1) - M_{fi}(eP_2) \qquad (12)$$

where

$$M_{fi}(P_1,P_2) = \frac{1}{(\overline{P-Q})^2}\int \phi_1^{s}(\mu,P-\mu)\,\phi_1^{s}(\mu,Q-\mu)\,d\mu\chi \times \int \phi_2^{0}(\sigma,Q'-\sigma)\,\phi_2^{s}(\sigma,P'-\sigma)\,d\sigma \tag{13}$$

$$M_{fi}(eP_1) = \frac{1}{(\overline{P-Q})^2}\int eP_1\,\phi_1^{s}(\lambda,P-\mu)\,\phi_1^{i}(\lambda,Q-\lambda)\,d\lambda \times \int \phi_2^{0}(\sigma,Q'-\sigma)\,\phi_2^{e}(P'-Q'+\sigma,\,Q'-\sigma)\,d\sigma \tag{14}$$

$$M_{fi}(eP_2) = \frac{1}{(\overline{P-Q})^2}\int \phi_1^{e}(\lambda,P-\lambda)\,\phi_1^{s}(\lambda-P+Q,\,P-\lambda)\,d\lambda \times \int d\sigma\,\phi_2^{e}(\sigma,P'-\sigma)\,\phi_2^{0}(\sigma,Q'-\sigma)\,d\sigma \tag{15}$$

where $\vec{Q}$, $\vec{Q}'$ are the incoming and $\vec{P}$, $\vec{P}'$ are the outgoing momenta of the two atoms. Then the inverse Fourier transform of (12) can be written as

$$M_{fi} = \phi_2^{0}(r_b)\,V\,\phi_2^{e}(r_b)\,dr_b dR \tag{16}$$

where V stands for the interaction potential, given by

$$V = \phi_1^{s}(r_a)\left[\frac{Z_1ZZ_1e^2}{R} + \frac{e^2}{|\,R+\bar{r}_a-\bar{r}_b\,|} - \frac{Z_1e^2}{|\,\bar{r}_a-R\,|} - \frac{Z_2e^2}{|\,r_b-R\,|}\right]\phi_1^{s}(r_a) \tag{17}$$

where $\bar{r}_a$, $\bar{r}_b$ etc. stand for same distances and co-ordinates as in BG.

Process of Second Kind

Following the same procedure as above we obtain the matrix element in the form as (17) except that $\phi_1^{s}(r_a)$ is replaced by $\phi_1^{e}(r_a)$, where superfix s denotes $1S$ state of the atom and 'e' any other excited state.

Conclusions

Our mode of approach is quite general and does not involve any type of ad-hoc assumption, so it seems that it can be applied to evaluate the interaction potential and cross-section for more complex systems.

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